

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1 1. (Currently amended) A method for bounding the solution set of a
2 system of linear equations $\mathbf{Ax} = \mathbf{b}$, wherein \mathbf{A} is an interval matrix and \mathbf{b} is an
3 interval vector, the method comprising:
4 receiving the system of linear equations $\mathbf{Ax} = \mathbf{b}$;
5 storing $\mathbf{Ax} = \mathbf{b}$ in a memory in a computer system;
6 preconditioning the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through
7 by a matrix \mathbf{B} to produce a preconditioned set of linear equations $\mathbf{BAx} = \mathbf{Bb}$,
8 wherein the set of linear equations is a representation of a global optimization
9 problem;
10 substituting $\mathbf{M}_0\mathbf{x} = \mathbf{r}$, wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ to produce $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
11 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
12 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
13 using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which bounds
14 the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$.

1 2. (Currently amended) The method of claim 1, wherein the method
2 further comprises computing the matrix \mathbf{B} by:
3 computing an approximate center \mathbf{A}_C of the interval elements of matrix \mathbf{A} ;
4 and
5 forming \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 3 (Canceled).

1 4. (Currently amended) The method of claim 1, further comprising
2 | assuring that $\sup(r_i) \geq 0$ by changing the sign of r_i $[[[]]$ and $x_i[[]]$ if necessary,
3 | wherein r_i is an element of \mathbf{r} .

1 5. (Original) The method of claim 1, further comprising:
2 determining if \mathbf{M} is regular; and
3 using the Gauss-Seidel process for computing the hull \mathbf{h} if \mathbf{M} is not
4 regular.

1 6. (Currently amended) A computer-readable storage medium storing
2 instructions that when executed by a computer cause the computer to perform a
3 method for bounding the solution set of a system of linear equations $\mathbf{Ax} = \mathbf{b}$,
4 wherein \mathbf{A} is an interval matrix and \mathbf{b} is an interval vector, the method
5 comprising:
6 receiving the system of linear equations $\mathbf{Ax} = \mathbf{b}$;
7 storing $\mathbf{Ax} = \mathbf{b}$ in a memory in a computer system;
8 preconditioning the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through
9 by a matrix \mathbf{B} to produce a preconditioned set of linear equations $\mathbf{BAx} = \mathbf{Bb}$.
10 wherein the set of linear equations is a representation of a global optimization
11 problem;
12 substituting $\mathbf{M}_0\mathbf{x} = \mathbf{r}$, wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ to produce $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
13 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
14 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
15 using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which bounds
16 the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$.

1 7. (Currently amended) The computer-readable storage medium of claim
2 6, wherein the method further comprises computing the matrix **B** by:
3 | computing an approximate center **A_C** of the interval elements of matrix A;
4 and
5 forming **B** by computing an approximate inverse of **A_C**, $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 8. (Currently amended) The computer-readable storage medium of claim
2 6, wherein using **M** and **r** to compute the hull **h** involves:
3 forming **P** as an inverse of the left endpoint of **M**;
4 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
5 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a unit vector in
6 | which the i -th element is 1 and other elements are 0, and wherein r_i is an element
7 | of **r**;
8 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$;
9 setting $\inf(h_i) = z_i$ if $z_i \leq 0$; and
10 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 9. (Currently amended) The computer-readable storage medium of claim
2 6, wherein the method further comprises assuring that $\sup(r_i) \geq 0$ by changing the
3 | sign of r_i [[[]] and x_i [[[]]] if necessary, wherein r_i is an element of **r**.

1 10. (Original) The computer-readable storage medium of claim 6, wherein
2 the method further comprises:
3 determining if **M** is regular; and
4 using the Gauss-Seidel process for computing the hull **h** if **M** is not
5 regular.

1 11. (Currently amended) An apparatus that bounds the solution set of a
2 system of linear equations $\mathbf{Ax} = \mathbf{b}$, wherein \mathbf{A} is an interval matrix and \mathbf{b} is an
3 interval vector, comprising:
4 a receiving mechanism configured to receive the system of linear
5 equations $\mathbf{Ax} = \mathbf{b}$;
6 a storing mechanism configured to store $\mathbf{Ax} = \mathbf{b}$ in a memory in a
7 computer system;
8 a preconditioning mechanism that is configured to precondition the set of
9 linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through by a matrix \mathbf{B} to produce a
10 preconditioned set of linear equations $\mathbf{BAx} = \mathbf{Bb}$, wherein the set of linear
11 equations is a representation of a global optimization problem;
12 substituting $\mathbf{M}_0\mathbf{x} = \mathbf{r}$, wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ to produce $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
13 a widening mechanism that is configured to widen the matrix \mathbf{M}_0 to
14 produce a widened matrix \mathbf{M} , wherein the midpoints of the interval elements of \mathbf{M}
15 form the identity matrix; and
16 a hull computing mechanism that is configured to use \mathbf{M} and \mathbf{r} to compute
17 the hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which bounds the solution set of the system
18 $\mathbf{M}_0\mathbf{x} = \mathbf{r}$.

1 12. (Currently amended) The apparatus of claim 11, wherein the
2 preconditioning mechanism is configured to:
3 compute an approximate center \mathbf{A}_C of the interval elements of matrix \mathbf{A} ;
4 and to
5 form \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 13. (Currently amended) The apparatus of claim 11, wherein the hull
2 computing mechanism is configured to:
3 form \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ;

4 form $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
5 form $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a unit vector in
6 which the i -th element is 1 and other elements are 0, and wherein r_i is an element
7 of \mathbf{r} ;
8 set $\inf(h_i) = c_i z_i$ if $z_i > 0$;
9 set $\inf(h_i) = z_i$ if $z_i \leq 0$; and to
10 set $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 14. (Currently amended) The apparatus of claim 11, wherein the
2 preconditioning mechanism is configured to assure that $\sup(r_i) \geq 0$ by changing
3 the sign of r_i ~~[[()]]~~ and x_i ~~[[()]]~~ if necessary, wherein r_i is an element of \mathbf{r} .

1 15. (Original) The apparatus of claim 11, wherein the preconditioning
2 mechanism is configured to:
3 determine if \mathbf{M} is regular; and to
4 terminate the process of computing the hull \mathbf{h} if \mathbf{M} is not regular.

1 16. (Currently amended) A method for bounding the solution set of a
2 system of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through by the matrix \mathbf{B} to
3 produce a preconditioned set of linear equations $\mathbf{BAx} = \mathbf{Bb}$, wherein the set of
4 linear equations is a representation of a global optimization problem, the method
5 comprising:
6 receiving the system of linear equations $\mathbf{Ax} = \mathbf{b}$;
7 storing $\mathbf{Ax} = \mathbf{b}$ in a memory in a computer system;
8 substituting $\mathbf{M}_0 \mathbf{x} = \mathbf{r}$, wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ producing $\mathbf{M}_0 \mathbf{x} = \mathbf{r}$;
9 ~~the method comprising:~~
10 assuring that $\sup(r_i) \geq 0$ by changing the sign of r_i (and x_i) if necessary;

11 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
 12 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
 13 using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} of the system $\mathbf{M}\mathbf{x} = \mathbf{r}$, which bounds
 14 the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ by,
 15 forming \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ,
 16 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$,
 17 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$,
 18 wherein e_i^T is a unit vector in which the i -th element is 1
 19 and other elements are 0, and wherein r_i is an element of \mathbf{r} ,
 20 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$,
 21 setting $\inf(h_i) = z_i$ if $z_i \leq 0$, and
 22 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 17. (Original) The method of claim 16, further comprising:
 2 determining if \mathbf{M} is regular; and
 3 using the Gauss-Seidel process for computing the hull \mathbf{h} if \mathbf{M} is not
 4 regular.

1 18. (Currently amended) The method of claim 16, wherein the method
 2 further comprises computing the matrix \mathbf{B} by:
 3 | computing an approximate center \mathbf{A}_C of the interval elements of matrix \mathbf{A} ;
 4 and
 5 forming \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.